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Competition among the big and the small

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**CORE**

DISCUSSION PAPER

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**Competition among the big and the small**

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**Abstract**

Many industries are made of a few big firms, which are able to manipulate the market outcome, and of a host of small businesses, each of which has a negligible impact on the market. We provide a general equilibrium framework that encapsulates both market structures. Due to the higher toughness of competition, the entry of big firms leads them to sell more through a market expansion effect generated by the shrinking of the monopolistically competitive fringe. Furthermore, social welfare increases with the number of big firms because the pro-competitive effect associated with entry dominates the resulting decrease in product diversity.

**Keywords:** oligopoly, monopolistic competition, product differentiation, welfare

**JEL Classification:** L13, L40

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# 1 Introduction

Many industries are polarized, involving a few large commercial or manufacturing firms, which are able to manipulate the market, and a host of small businesses, each of which has a negligible impact on the market. Examples can be found in apparel, catering, publishers and bookstores, retailing, finance and insurances, hotels, and IT industries. Business scholars such as Porter (1982) stress the fact that firms within the same industry are often clustered in groups with distinct business models and operations. The same holds in international trade, where a few large firms account for the bulk of exports (Bernard et al., 2007). Standard theories of imperfect competition, which are split between oligopoly and monopolistic competition models, do not reflect the nature of such *mixed markets*. The reason is that these markets blend a small number of large incumbents, which behave strategically, and a monopolistically competitive fringe, in which firms maximize their profits on their residual demand in the absence of strategic interactions.

The purpose of this article is to develop a unified framework to study (i) how large and small firms interact to shape the market outcome and (ii) whether or not it is socially desirable to have large and/or small firms in business. To reach our goal, we combine two standard models of imperfect competition, namely the oligopoly model à la Cournot with symmetrically differentiated products (Vives, 1999) and the monopolistic competition model developed by Dixit and Stiglitz (1977). Specifically, we assume that big firms behave strategically and manipulate market aggregates such as the price index, whereas small firms accurately treat these market aggregates parametrically because they are negligible. This modeling strategy agrees with Aumann (1964), who suggests to combine a continuum of traders and a few large traders to study market power. Moreover, large and small firms choose their output simultaneously. In addition, although there is a continual flow of firms entering or exiting the market, this process seldom undermines the big firms' position. Consequently, we assume that the mass of monopolistically competitive firms adjust to the number of large firms through the usual process of free entry and exit described in monopolistic competition. By contrast, the entry of large firms is exogenous.

Our main findings are as follows. First of all, the entry of a large firm generates two opposing effects. On one hand, as in standard oligopoly theory, entry tends to depress the large incumbents' output. On the other hand, by making competition fiercer, the entry of a large firm leads to a shrinking of the monopolistically competitive fringe.<sup>1</sup> This in turn triggers a market expansion effect that fosters an increase of the large incumbents' output. A priori, the net impact of entry seems to be ambiguous. Our analysis reveals that entry leads to an unambiguous increase in the output of every large firm.<sup>2</sup> Furthermore, the entry of a big firm leads to a decrease in the industry price index

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<sup>1</sup>Note that there has been in the UK a sharp decline in the number of small groceries after the passage of the Resale Prices Act in 1964 abolishing resale price maintenance (Everton, 1993).

<sup>2</sup>Note that oligopoly theory has identified settings in which entry triggers a price hike; see Chen and Riordan (2008)

and to an increase in the output of the industry as a whole. In a nutshell, *the addition of a large firm to a market is more powerful in promoting competition than the preservation of small firms*. Conversely, restricting the entry of large producers allows a whole range of small firms to survive but makes the market less competitive.

Second, because of the above-mentioned market expansion effect, when entry arises under the concrete form of a new large firm, the exit of a range of small firms allows the large firms to earn higher profits. Again, this is to be contrasted with the oligopoly case where entry lowers profits. Note the following general equilibrium effect: higher profits lead to a higher total income, which shifts upward the demand of both large and small firms and generates a richer set of interactions among firms. Lastly, in terms of welfare, we show the unexpected (at least to us) result that, *despite the diversity reduction caused by the exit of small firms, the entry of a big firm is beneficial to consumers*. It is worth noting that those results are obtained without making specific assumptions about firms' marginal costs. The only assumption is that these parameters are such that both kinds of firms coexist. Thus, we may safely conclude that *the mixed market structure differs in several respects from the oligopoly setting*.

Our analysis also has some competition policy implications which are worth mentioning. Several countries have passed bills that restrict the entry of large firms or the expansion of existing ones, by forbidding price discounts or regulating the hours of operations in order to permit small firms to remain active. To illustrate, consider the case of the retailing sector, which has attracted a lot of attention in several countries. In France, the Royer-Raffarin Law imposes severe restrictions on the entry of department stores whose surface exceeds 300 square meters, the justification being that small shops provide various convenience services. It is worth mentioning here that Bertrand and Kramarz (2002) show that the enforcement of the Royer-Raffarin Law has had a negative impact on job creation in France. This in turn suggests that this regulation has lowered the output and increased the price index of the French retail sector, as suggested by our model. The Net Book Agreement in the United Kingdom between book publishers and retailers forbids discounts on books with the aim of preserving a large network of small bookstores, whereas in France the Lang Law, which also prevents price discounting, is argued by the publishers and small book sellers to be justifiable on the same grounds. In the case of Japan, Garon and Mochizuki (1993) argue that small-business associations aim to exchange their political influence for governmental policies that compensate for their weakness in the marketplace.

Even though the objective of such laws and regulations was often to gain the political support of small-business associations, popular thinking in developed countries has it that small firms allow for a wider array of varieties and services. We find it fair to say, however, that the public often dismisses the

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and the references contained therein. However, the reason for price-increasing competition identified by Chen and Riordan are very different from ours.

fact that the presence of large retailers fosters lower prices than small ones, thus allowing households to increase their consumption (Basker, 2007). Our analysis confirms that deregulating mixed markets causes the progressive disappearance of small firms. However, by showing that welfare increases with the entry of big firms, it casts doubt on the economic foundations of the various laws and regulations that tend to keep active a large number of small businesses.

The issue addressed in this article is related to, but different from, several existing contributions. First, in the dominant firm model, one large firm and a competitive fringe coexist (Markham, 1951). Our setting markedly differs from this model. First, it does not capture the above-mentioned diversity effect because all firms produce the same homogeneous good. Second, the dominant firm is the leader of a Stackelberg game in which the small firms are the followers. In contrast, here all firms play simultaneously and supply differentiated varieties. There are some similarities, however. In the dominant firm model, the small firms face an increasing marginal costs and a given price; in the Dixit-Stiglitz model of monopolistic competition, the small firms face a decreasing marginal revenue and a given marginal cost. Our analysis differs from Chen (2003) as well as from Gowrisankaran and Holmes (2004), who use the dominant firm model to study questions different from ours. Holmes (1996) also uses the dominant firm model and deals with issues that are related to what we do in this article. In particular, he shows that restricting the size of the dominant firm is detrimental to consumers in the case where the dominant and fringe firms have the same technology. Note that our results hold in the absence of such restrictions.

Another related contribution is Gabszewicz and Vial (1972), who study the Cournot–Walras model in which firms first select quantities, while market prices are established at the Walrasian equilibrium of the resulting exchange economy. Using the so-obtained demands, firms choose their outputs at the Cournot equilibrium. In doing so, firms are aware that they manipulate consumers’ demand functions through the redistribution of profits. The main issue encountered with this family of models is the frequent non-existence of an equilibrium (Bonanno, 1990). One possible way out is considered by Neary (2009), who assumes a continuum of sectors, each being endowed with a small number of strategic firms. In this case, each firm has a positive impact on its competitors, but no impact on the economy as a whole because each sector is negligible. Thus, profits earned by firms belonging to the same sector have no impact on these firms’ demands. The total income effect affects firms only through the marginal utility of income. Lastly, while all the above contributions are cast within the framework of noncooperative game theory, a few contributions have studied the interactions between big and small traders in an exchange economy, using cooperative game theory (Gabszewicz and Shitovicz, 1992).

The model is described in detail in the next section. Section 3 shows the existence of a mixed market equilibrium and studies its main properties. Because big and small firms have different market behavior, we have not been able to derive explicit solutions, which means that our analysis

is conducted through implicit expressions. The welfare analysis is taken up in Section 4. Section 5 concludes. Proofs are given in the appendix.

## 2 The model

### Preferences and demand

The economy involves two goods, two sectors, and one production factor - labor - which is mobile between sectors. The first good is a horizontally differentiated good; it is produced under increasing returns and supplied both by oligopolistic and monopolistically competitive firms (MC-firms). The second good, which accounts for the rest of the economy, is homogeneous and produced under constant returns to scale and perfect competition.

The first issue that we must address is how to model the large and small firms operating in the differentiated sector. We assume that there are  $N$  large firms having a positive measure and a mass  $M > 0$  of small firms having a zero measure. Consequently, each large firm affects the market whereas each small firm is negligible to the market. Thus, in our setting large and small firms differ in *kind* unlike Melitz (2003) where all firms are infinitesimal in scale. The number  $N$  of large firms is exogenous but the size  $M$  of the monopolistically competitive fringe is endogenous. For our setting to account for oligopolistic competition, we assume that  $N \geq 2$ . That said, we now describe how preferences are defined over the set of varieties.

By convention, variables associated with large firms are denoted by capital letters and those corresponding to small firms by lower case letters. The field of monopolistic competition being dominated by the CES, we assume that the differentiated good is formed by two CES-composite goods,  $Q_0$  and  $\mathbf{Q}_1$ , defined as follows:

$$Q_0 = \left( \int_0^M q_i^\rho di \right)^{\frac{1}{\rho}} \quad \mathbf{Q}_1 = \left( \sum_{j=1}^N Q_j^\rho \right)^{\frac{1}{\rho}} \quad (1)$$

where  $q_i$  is the output level of the small firm  $i \in [0, M]$ ,  $Q_j$  the output level of the large firm  $j = 1, \dots, N$  and  $0 < \rho < 1$  a given parameter. In the Dixit-Stiglitz model of monopolistic competition, consumers' utility depends only upon  $Q_0$ . By contrast, in oligopoly only  $\mathbf{Q}_1$  matters to consumers. Our aim being to combine both types of competition, we aggregate the two composite goods in the following way:

$$\mathbf{Q} = (Q_0^\rho + \mathbf{Q}_1^\rho)^{\frac{1}{\rho}} \quad (2)$$

where  $\mathbf{Q}$  is the output index of the entire differentiated sector.

The asymmetric treatment of the large and small firm's outputs,  $Q_i$  and  $q_i$ , and the aggregation in (2), may be justified in the following way. Consider two differentiated goods,  $k = 0, 1$  produced

by two types of firms. Denoting by  $Q_k$  the output of a type  $k$ -firm and by  $N_k$  the number of such firms, the CES-composite good is given by

$$\mathbf{Q} = (w_0 N_0 Q_0^\rho + w_1 N_1 Q_1^\rho)^{1/\rho}$$

where  $w_k$  is the preference parameter associated with good  $k = 0, 1$ . By changing the relative value of  $w_0$  and  $w_1$ , we change the demands for each type of good and, therefore, the market outcome. To be precise, as  $w_0$  steadily decreases with respect to  $w_1$ , the equilibrium output of a firm producing good 1 grows while the equilibrium output of a firm producing good 0 shrinks. In this context, (2) may be viewed as the limiting case in which the number  $N_1$  of type 1-firms and the parameter  $w_1 > 0$  are given, whereas the preference parameter  $w_0$  tends to 0 and the number  $N_0$  of type 0-firms becomes arbitrarily large.

Alternatively, we could follow Neary (2010) and Parenti (2010), who propose to model large firms as producers supplying a continuum of varieties, whereas each small firm supplies a single variety.<sup>3</sup> With single-product firms, diversity in the industry is determined by a trade-off between the cost of introducing a new variety in the market and the associated revenues. In this case, there are no scope economies and entrants do not internalize the business stealing effect they have on other firms. With multiproduct firms, the trade-off is more complex due to the presence of scope economies and the internalization of the business stealing effect among the varieties launched by the same firm (cannibalization). Parenti (2010) shows how to deal with this issue in the case of a quadratic subutility nested into a linear utility.

When the product range is exogenous, the above approach does not differ from that proposed in this article because the coefficient  $w_1$  may be reinterpreted as the breadth of the large firms' product range. To be precise,  $Q_j$  is now the CES-composite good of the varieties supplied by the large firm  $j$ . The coefficient  $w_1$  can be normalized to 1 by choosing appropriately the unit of the real line along which the mass of varieties is measured. This means that the breadth of the product range has no impact on our results. Put differently, *how wide is the product range provided by the big firms does not matter for our results*. Note, however, that (2) imposes that the length of the product range is fixed and the same across large firms. Therefore, our approach takes into account the business stealing effect but not the cannibalization issue.

Because a consumer endowed with CES preferences may represent a large population of heterogeneous consumers, we simplify notation by assuming that the demand side is described by a representative consumer (Anderson et al., 1992). This agent is endowed with  $L$  units of labor, holds

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<sup>3</sup>Neary (2010) suggests a third approach in which firms first enter the market, and then choose to become large or to remain small. In the last stage, all firms are of a specific kind and compete on the market as they do in this article. In contrast, we assume here that firms are born big or small.

the shares of all firms, and has a preference relation represented by the following utility function:

$$U = \mathbf{Q}^\alpha \cdot X^{1-\alpha} = \left( \sum_{j=0}^N Q_j^\rho \right)^{\alpha/\rho} \cdot X^{1-\alpha} \quad (3)$$

where the industry output index  $\mathbf{Q}$  is given by (2), while  $X$  is the consumption of the homogeneous good and  $\alpha$  a given parameter satisfying the inequality  $0 < \alpha < 1$ .

The upper-tier utility being of the Cobb-Douglas type, the homogeneous good is always produced and consumed. Without loss of generality, we assume that one unit of labor produces one unit of the homogeneous good. We choose this good as the numéraire. Therefore, the equilibrium wage is equal to 1. Our primary purpose being to investigate how large and small firms interact on the product market, assuming that workers' wage is given allows us to isolate this effect from other considerations such as the working of the labor market.

Observe that the process of substitution between the two kinds of goods is more involved than in standard oligopoly or monopolistic competitive models. To illustrate how it works, consider the situation in which the quantities  $Q_j$  are the same and equal to  $Q$ , whereas the output density  $q_i$  is uniform and equal to  $q$ . If an additional variety  $N + 1$  becomes available in quantity  $Q$ , the total mass of negligible varieties that leaves the utility level unchanged must decrease by  $\Delta M = (Q/q)^\rho$ . In other words, the entry of variety  $N + 1$  triggers the exit of a positive range of varieties supplied by the MC-subsector.

The representative consumer maximizes her utility subject to

$$\sum_{j=1}^N P_j Q_j + \int_0^M p_i q_i di + X = \mathbf{Y}$$

where  $P_j$  is the price of variety  $j = 1, \dots, n$ ,  $p_i$  the price of variety  $i \in [0, M]$ , and  $\mathbf{Y}$  the income level given by the wage bill  $L$  plus profits. Note that the value of the income  $\mathbf{Y}$  is endogenous because profits are determined at the equilibrium. The income share spent on the differentiated good being constant, we set  $\mathbf{y} \equiv \alpha \mathbf{Y}$ .

It is well known that the price index of the MC-subsector is given by

$$P_0 \equiv \left( \int_0^M p_i^{\frac{-\rho}{1-\rho}} di \right)^{-\frac{1-\rho}{\rho}} \quad (4)$$

and thus the industry price index  $\mathbf{P}$  is

$$\mathbf{P} \equiv \left( \sum_{j=0}^N P_j^{\frac{-\rho}{1-\rho}} \right)^{-\frac{1-\rho}{\rho}}. \quad (5)$$

Clearly, the industry price index increases with the price  $P_j$  of any variety  $j$  as well as with the price index  $P_0$  of the MC-subsector.



The inverse demand functions are given by

$$P_j(Q_j, \mathbf{P}, \mathbf{y}) \equiv \mathbf{y}^{1-\rho} Q_j^{-(1-\rho)} \mathbf{P}^\rho \quad j = 1, \dots, N \quad (6)$$

$$p_i(q_i, \mathbf{P}, \mathbf{y}) = \mathbf{y}^{1-\rho} q_i^{-(1-\rho)} \mathbf{P}^\rho \quad i \in [0, M]. \quad (7)$$

Hence, small firms face demands having the same constant price-elasticity, whereas large firms' demands displays a *variable* price-elasticity because  $\mathbf{P}$  changes with  $Q_j$ . Furthermore, holding  $\mathbf{y}$  constant, both demands are decreasing in their own output, while  $\partial P_j / \partial Q_k > 0$  for  $k \neq j$ .

Substituting (6) and (7) into (5) yields the industry price index as a function of the industry output index and income:

$$\mathbf{P} = \mathbf{y} \mathbf{Q}^{-1}. \quad (8)$$

## Large firms

It follows from (6) that the profits of the large firm  $j$  is given by

$$\Pi_j(Q_j, \mathbf{P}, \mathbf{y}) = [P(Q_j, \mathbf{P}, \mathbf{y}) - C]Q_j - F = \mathbf{y}^{1-\rho} Q_j^\rho \mathbf{P}^\rho - CQ_j - F \quad (9)$$

where  $C > 0$  is the constant marginal cost and  $F$  the fixed cost. Note that fixed costs do not play any role in Section 3. They are needed for the welfare analysis conducted in Section 4.

Any large firm is aware that its output choice affects the industry price index  $\mathbf{P}$  and is, therefore, involved in a game-theoretic environment. It also understands that  $\mathbf{P}$  is influenced by the aggregate behavior of the MC-firms expressed by  $Q_0$ .<sup>4</sup> Last, as shown by (6) and (7), the income level influences firms' demands, whence their profits. As a result, all firms must anticipate correctly what the total income will be.

Because they have a positive measure, the large firms should be aware that they can manipulate the income level, whence their demands, through their output choices (the Ford effect). However, accounting for such feedback effects often leads to the nonexistence of an equilibrium, the reason being that profit functions are not quasi-concave (Roberts and Sonnenschein, 1977).<sup>5</sup>

In what follows, we consider a different approach and assume that large firms treats  $\mathbf{y}$  parametrically. In other words, large firms behave like *income-takers*.<sup>6</sup> This approach is in the spirit of Hart (1985) for whom firms should take into account only some effects of their policy on the whole

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<sup>4</sup>Because the upper-tier utility is of the Cobb-Douglas type, the manipulation of the price index has no impact on the income share spent on the differentiated product.

<sup>5</sup>A noticeable exception is d'Aspremont *et al.* (1996), who take the Ford effect into account and solve the general equilibrium CES model with oligopolistic firms. However, unlike ours their setting is symmetric. This vastly simplifies their analysis.

<sup>6</sup>The same difficulty arises when governments, clubs or developers providing a public good manipulate strategically the utility level. The corresponding public economics literature thus relies on the assumption that these big agents are utility-takers (Scotchmer, 2002).

economy. It also concurs with Neary (2009) when the sector under consideration is small ( $\alpha$  is close to 0) or when each large firm within its sector is small in the economy as a whole. Note that the income-taking assumption does not mean that profits have no macroeconomic impact. It means that no large firm seeks to manipulate its own demand through the income level, which seems reasonable in large and diversified economies (recall that the total wages paid by the large firms are taken into account in  $L$ ).

Accordingly, although our model does not capture all feedback effects, it is a full-fledged general equilibrium model in which large firms account for (i) strategic interactions within their group, (ii) the aggregate behavior of the small firms, and (iii) the endogenous income generated by profit distribution. In other words, our model is *not* a partial equilibrium one, the difference being that the income level is exogenous in a partial equilibrium model whereas it is endogenous here.

Let  $Q_{-j} \equiv (Q_1, \dots, Q_{j-1}, Q_{j+1}, \dots, Q_N)$  be the vector of all outputs but that of firm  $j$ . Because  $\partial \Pi_j / \partial Q_j$  is strictly decreasing in  $Q_j$ , we have:

**Lemma 1.** *For any  $j = 1, \dots, N$  and any given  $Q_{-j}$  and  $Q_0$ ,  $\Pi_j$  is strictly concave with respect to  $Q_j$ .*

Hence, firm  $j$ 's best reply  $Q_j^*(Q_{-j}, Q_0; \mathbf{y})$  is the unique solution to the first-order condition:

$$\frac{\partial \Pi_j}{\partial Q_j} = \frac{\rho \sum_{k \neq j} Q_k^\rho}{Q_j^{1-\rho} \left( Q_j^\rho + \sum_{k \neq j} Q_k^\rho \right)^2} \mathbf{y} - C = 0. \quad (10)$$

## Small firms

Being infinitesimal in scale every small firm accurately treats the industry price index and the income as given parameters. The difference in firms' behavior reflects the difference in the underlying market structure that characterizes each subsector.

The profit of the small firm  $i \in [0, M]$  is given by

$$\pi_i(q_i; \mathbf{P}, \mathbf{y}) = \mathbf{y}^{1-\rho} q_i^\rho \mathbf{P}^\rho - cq_i - f$$

where  $c > 0$  is the constant marginal cost and  $f > 0$  the fixed cost. Observe that large and small firms are homogeneous within their own group but heterogeneous between groups.

Because  $\rho < 1$ ,  $\pi_i(q_i; \mathbf{P}, \mathbf{y})$  is strictly concave in  $q_i$ . Applying the first-order condition yields the equilibrium price of a small firm

$$p^* = \frac{c}{\rho} \quad (11)$$

which is the same as the price prevailing under monopolistic competition. By contrast, a small firm's equilibrium output

$$q^* = \left( \frac{c}{\rho} \right)^{\frac{-1}{1-\rho}} \mathbf{y} \mathbf{P}^{\frac{\rho}{1-\rho}} \quad (12)$$

varies with the quantities chosen by the large firms through the price index  $\mathbf{P}$  and the income  $\mathbf{y}$ .

Substituting (12) into (4) and (1) in (11), we obtain:

$$Q_0 = \left(\frac{c}{\rho}\right)^{\frac{-1}{1-\rho}} M^{\frac{1}{\rho}} \mathbf{y} \mathbf{P}^{\frac{\rho}{1-\rho}} \quad (13)$$

$$P_0 = \frac{c}{\rho} M^{-\frac{1-\rho}{\rho}}. \quad (14)$$

In words, the price index  $P_0$  of the monopolistically competitive fringe depends only upon its size: the larger  $M$ , the lower  $P_0$ . Although the equilibrium price of each variety is independent of  $M$ , (14) implies that a larger mass of small firms makes competition tougher through more fragmented individual demands, thus leading to a lower price index  $P_0$ . This shows how the size of the monopolistically competitive fringe affects the intensity of competition in the whole industry.

For any given  $M$  and  $N$ , (12) also implies that the equilibrium profit of a small firm is given by

$$\pi^*(\mathbf{P}, \mathbf{y}) = (1 - \rho) \left(\frac{c}{\rho}\right)^{\frac{-\rho}{1-\rho}} \mathbf{y} \mathbf{P}^{\frac{\rho}{1-\rho}} - f. \quad (15)$$

### 3 The market outcome

We consider a non-cooperative game in which big and small firms choose their output simultaneously. The *market equilibrium* is defined as a state in which the following conditions hold: (i) the representative consumer maximizes her utility subject to the budget constraint, (ii) both large and small firms maximize their own profits with respect to output, (iii) large firms earn positive profits, (iv) the mass of MC-firms is adjusted until profits are zero or no MC-firm operates:

$$\begin{aligned} M^* > 0 &\Rightarrow \pi^* = 0 \\ \pi^* < 0 &\Rightarrow M^* = 0 \end{aligned}$$

and (v) all markets clear. When  $N \geq 2$  and  $M^* > 0$ , we say that the market equilibrium is *mixed*.

For any given  $N \geq 2$ , we may characterize the mixed market equilibrium by means of the following four conditions: (i) the profit-maximization conditions of small firms, (ii) the profit-maximization conditions of large firms, and (iii) the zero-profit condition for small firms. In this way, we consider 0 as a “pseudo-player” who chooses the mass of small firms non-strategically.

#### Existence of a mixed market equilibrium

Consider a mixed market equilibrium in which the large firms choose the same output  $Q$  sold at the same price  $P$ , whereas the small firms produce the same output  $q$ .<sup>7</sup> Hence, symmetry prevails within

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<sup>7</sup>Assume that a mixed market equilibrium exists. Then, given the corresponding values of  $\mathbf{Y}^*$  and  $Q_0^*$ , the large firms always choose the same output  $Q^*$  (Vives, 1999).

each group of firms but not between groups.

Our analysis involves two steps: (i) we calculate the equilibrium conditions when the size  $M$  of the MC-subsector is fixed and (ii) we determine the equilibrium value of  $M$ .

**Step 1.** The total income  $\mathbf{Y}$  is implicitly given by

$$\mathbf{Y} = L + N\Pi(Q, Q_0, \mathbf{P}; \mathbf{y}) + M\pi^*(\mathbf{P}, \mathbf{y}). \quad (16)$$

Furthermore, (8) implies

$$\mathbf{P} = \mathbf{y} (Q_0^\rho + NQ^\rho)^{-\frac{1}{\rho}}. \quad (17)$$

Using this expression, we can rewrite the first-order condition (10) as follows:

$$\mathbf{y}^{1-\rho} = \frac{C}{\rho} \mathbf{P}^{-\rho} Q^{1-\rho} + \mathbf{y}^{1-2\rho} \mathbf{P}^\rho Q^\rho. \quad (18)$$

The four equations (13), (16), (17) and (18) yield the equilibrium values of  $Q_0(M)$ ,  $Q(M)$ ,  $\mathbf{y}(M)$  and  $\mathbf{P}(M)$ . Plugging  $\mathbf{y}(M)$  and  $\mathbf{P}(M)$  into  $\pi^*(\mathbf{P}, \mathbf{y})$ , we obtain the profit function  $\pi^*(M)$  in terms of  $M$  only.

We start with the following result, the proof of which is given in Appendix A.

**Lemma 2.** *For any given value of  $N$ , the equilibrium profit of an MC-firm  $\pi^*(M)$  is a strictly decreasing function of  $M$ .*

Thus, the entry and exit process of small firms yields a unique and stable solution  $M^*$  to the zero-profit condition  $\pi^*(M) = 0$ . Furthermore, Lemma 2 and (15) imply that the monopolistically competitive fringe shrinks when the fixed cost  $f$  rises.

We now come to the impact of  $M$  on prices. We already know that  $P_0$  decreases with  $M$ . The impact of  $M$  on the common price charged by the large firms and on the industry price index is less straightforward. In Steps 1 and 2 of Appendix A, we show that increasing  $M$  leads to lower values for  $P(M)$  and  $\mathbf{P}(M)$ . This is because the entry of small firms intensifies competition between the two groups of firms, which in turn strengthens competition *within* the group of large firms and results in a lower price  $P$ .

To sum-up, we have:

**Proposition 1** *Assume that the size of the MC-subsector is exogenous. Then, both the industry price index and the price at which the large firms sell their output decrease when the mass of small firms increases.*

Therefore, the market reacts as if the monopolistically competitive fringe were a single big firm producing  $Q_0$ . This confirms the idea that the MC-subsector may be viewed as a pseudo-player. It should be kept in mind, however, that  $Q_0$  is not the output chosen by this pseudo-player. It stems

from the aggregation of production decisions made by a myriad of small firms. Note also that we do not know yet how  $Q_0$  varies with  $M$ .

**Step 2.** Using (15), the zero-profit condition  $\pi^* = 0$  is equivalent to

$$\mathbf{y} = \frac{f}{1-\rho} \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} \mathbf{P}^{\frac{-\rho}{1-\rho}}. \quad (19)$$

Hence, under free entry in the MC-subsector the equilibrium values of  $\mathbf{y}$  and  $\mathbf{P}$  are inversely related.

Plugging (19) into (12), we obtain

$$q^* = \frac{\rho f}{(1-\rho)c}$$

which is identical to the equilibrium size of a firm under monopolistic competition.

Therefore,

$$Q_0^* = \frac{\rho f}{(1-\rho)c} (M^*)^{\frac{1}{\rho}}. \quad (20)$$

To put it simply, under free entry the output index of the monopolistically competitive fringe is determined by the sole mass of MC-firms. Consequently, *the small firms adjust to market changes through entry or exit only.*

Last, it follows from (8) and (19) that the industry price index  $\mathbf{P}$  decreases with the industry output index  $\mathbf{Q}$ . Hence, under free entry and variable income, the downward sloping relationship between price and quantity holds at the aggregate level.

The mixed market equilibrium  $(Q^*, \mathbf{P}^*, \mathbf{y}^*, M^*)$  is determined by the four conditions (16)-(19). Because there are two kinds of firms whose market behavior differs, showing the existence of an equilibrium not standard. Furthermore, it should be clear that restrictions on the parameters must be imposed for a mixed market equilibrium to exist. If the fixed cost  $f$  ( $F$ ) is high relative to the market size  $L$ , no small (large) firm operate. Therefore, we must find the conditions under which the two kinds of firms are active in equilibrium.

In Appendix B.1, we show that the market outcome involves a monopolistically competitive fringe if and only if  $f/L < S(N; F/L)$ , where  $S$  is the curve describing the set of parameters  $N$  and  $F/L$  such that the MC-subsector just vanishes in equilibrium. As expected, high values of  $f/L$  prevent the existence of a monopolistically competitive fringe. The function  $S$  is linear and downward sloping in  $F/L$ , with  $S(N; 0) > 0$  and  $S(N; F/L) \rightarrow 0$  when  $F/L \rightarrow 1/N$ . Indeed, as the number of large firms grows, small firms are gradually driven out of business.

In Appendix B.2, we establish that  $N$  large firms' profits are positive in equilibrium if and only if  $B(F/L) < f/L$ , where  $B$  is the locus where these profits are just equal to zero. The function  $B$  is strictly increasing, with  $B(0) = 0$  and  $B(F/L) \rightarrow \infty$  when  $F/L \rightarrow (1-\alpha)\rho$ . As expected, high values of  $F/L$  prevent big firms to be active at the market outcome.

Accordingly, the domain of the  $(F/L, f/L)$ -plane for which a mixed market equilibrium prevails is defined by the intersection of the two sets delineated by  $B(F/L) = f/L$  and  $f/L = S(N; F/L)$ .

It is non-empty because  $S$  is strictly decreasing with  $S(N; 0) > 0$ , while  $B$  is strictly increasing with  $B(0) = 0$ . Consequently, we have:

**Proposition 2** *For any given  $N$  such that  $2 \leq N < L/F$ , there exists a unique mixed market equilibrium if and only if*

$$B\left(\frac{F}{L}\right) < \frac{f}{L} < S\left(N; \frac{F}{L}\right).$$

In Figure 1, we depict the domain of parameters in which such a mixed market equilibrium exists. Depending on the relative values of  $F/L$  and  $f/L$ , the economy may have a handful of big firms and/or a myriad of small firms. In particular, increasing the value of  $f/L$  leads to the widening of the range of  $(F/L)$ -values for which the market involves large firms only. This is because it becomes harder for small firms to survive. In contrast, when  $F/L$  increases, the range of  $(f/L)$ -values for which the market involves small shrinks. As shown below, this is caused by an income effect that stems from the general equilibrium nature of our setting.

Insert Figure 1 about here

Because  $S$  is decreasing in  $N \geq 2$ , arbitrarily large at  $N = 1$ , and negative when  $N$  tends to infinity, the equation

$$S\left(N; \frac{F}{L}\right) = \frac{f}{L}$$

has a unique solution  $\bar{N}$ . In other words,  $M^* = 0$  and  $Q_0^* = 0$  when  $N$  is larger than or equal to  $\bar{N}$ . As to be expected, when the level of fixed costs in the MC-subsector gets lower, more big firms are needed to trigger the disappearance of the monopolistically competitive fringe.

Figure 2 shows how the equilibrium values of  $Q$  and  $\mathbf{y}$  are determined at the intersection of two curves (see Appendix B.2 for more details). The former describes the relationship (16), which gives the equilibrium value of  $\mathbf{Y}$  when the large firms produce  $Q$ :

$$\mathbf{Y} = L + N\Pi(Q). \quad (21)$$

The latter is obtained by combining two other equilibrium conditions. Solving the profit-maximizing condition (18) for  $\mathbf{P}$  and plugging the resulting expression into the zero-profit condition (19) leads to the condition

$$1 = \frac{C}{\gamma\rho}Q^{1-\rho} + \mathbf{y}^{-1}\gamma Q^\rho$$

which relates  $Q$  and  $\mathbf{y}$  at the equilibrium;  $\gamma > 0$  is a bundle of parameters (see Appendix B.2). Solving this expression with respect to  $\mathbf{y}$ , we obtain

$$\mathbf{Y} = \frac{\gamma}{\alpha} \frac{Q^\rho}{1 - (C/\gamma\rho)Q^{1-\rho}} \equiv \Phi(Q) \quad (22)$$

which defines a second relationship between the equilibrium values of  $Q$  and  $\mathbf{Y}$ . In words, the equation (22) gives the large firms' profit-maximizing output when these firms expect the total income to be equal to any given value. By construction, the two curves (21) and (22) intersect at the equilibrium values of  $\mathbf{Y}$  and  $Q$ . Figure 2 shows that, for any value of  $N$ , these curves intersect only once.

Insert Figure 2 about here

## The industry structure

The aim of this subsection is to study how the two subsectors are affected by the entry of a large firm. Our first two results highlight how the two subsectors react to the addition of a big competitor.

(i) When the number of large firms rises, Figure 2 shows that the curve (21) is shifted upward. By contrast, the curve (22) is unaffected. As a result, when the number of large firms increases from  $N_1$  to  $N_2$ , the equilibrium output rises from  $Q_1^*$  to  $Q_2^*$ .

**Proposition 3** *In a mixed market, the entry of a large firm leads the large incumbents to raise their output.*

It seems natural to ask whether Proposition 3 is due to the mixed nature of the market or to the income effect generated by the redistribution of profits? To answer this question, we isolate the income effect by considering the impact of entry in a market involving only oligopolistic firms. In this case, as shown by (B.12), firms' output is given by

$$Q^O = \frac{\alpha\rho}{C} \frac{(N-1)(L-NF)}{N[(1-\alpha+\alpha\rho)N-\alpha\rho]}.$$

Differentiating this expression with respect to  $N$  for  $N \geq 2$  shows that  $Q^O$  decreases with  $N$ . Accordingly, we need a mixed market structure for the output growth effect to occur.

(ii) We now show how the monopolistically competitive fringe reacts to the entry of a large firm. Using the expression of  $\pi^*(M)$  given by (A.9) in Appendix A, it is readily seen that  $\pi^*(M)$  decreases with  $M$  for any given  $N$  while  $\pi^*(M)$  is shifted downward when  $N$  increases. Therefore, the equilibrium mass of small firms  $M^*(N)$  must decrease with  $N$ .

**Proposition 4** *In a mixed market, the entry of a large firm leads to a shrinking of the monopolistically competitive fringe.*

This result is in accordance with Basker (2007) who observes that, in the U.S. retail sector, Wal-Mart's competitive pressure has caused other stores, especially small ones, to shut down. Disregarding its productive advantage, it suggests that the entry of Wal-Mart should have increased the sales

of Target, that is, the second-largest discount retailer in the United States, at the expense of small retailers.

The above two propositions may be combined to describe the main forces at work in a mixed market. *By contracting the monopolistically competitive fringe, the entry of a large producer triggers a market expansion effect that allows the large incumbents to increase their output.* For this market expansion effect to arise there must be a monopolistically competitive fringe that acts as a buffer. This reveals the existence of a trade-off between the two subsectors: when one subsector grows, the other declines (see Proposition 1).

The intuition behind Proposition 3 is now clear. The small firms have a strategic advantage in dealing with the large ones because they do not take into account the impact of their output decisions on the industry price index. This enables the small firms to commit to a larger output than they would if the MC-subsector acted as a group. Indeed, in this case we would be back to a pure oligopolistic world in which entry leads the incumbents to contract their outputs (see B.12). Simply put, as small firms gradually exit the market, the large firms take advantage of the disappearance of such “aggressive” competitors to expand their output.<sup>8</sup>

We need two more properties of the equilibrium to complete our study of the interactions between the two kinds of firms.

(iii) When the number of large firms increases from  $N_1$  to  $N_2$ , Figure 2 shows that the equilibrium income from  $\mathbf{Y}_1^*$  to  $\mathbf{Y}_2^*$ . Because the functions  $L + N\Pi(Q)$  and  $\Pi(Q)$  behave alike and because  $Q^*(N)$  increases with  $N$  over the interval  $]0, \bar{Q}[$ , it follows immediately that:

**Proposition 5** *In a mixed market, the entry of a large firm raises the profit of each large incumbent.*

This unsuspected result is the outcome of the interplay between several intertwined effects. First, as seen above, when a new firm enters the market, the mass of small firms decreases, which generates a market expansion effect that allows the big firms to expand their output and profits. This is to be contrasted with the oligopoly case in which individual output and profits decrease because the large firms do not benefit from the above market expansion effect. Furthermore, higher profits result in a higher income which fuels the expansion of the market for *each* kind of firms. All else equal, this allows a larger number of small firms to stay in business. Even though this effect slows down the exit of small firms (see Proposition 1), it is not sufficiently strong to break it off.

The role of the income effect is highlighted by assuming that profits are redistributed to absentee shareholders. In this case, the curve  $\mathbf{Y} = L$  is flat, and thus the equilibrium output  $Q^*$  is unaffected by entry. However, the market expansion effect is still at work in such a partial equilibrium setting because the output produced in the oligopoly case decreases with the addition of a large firm. Yet,

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<sup>8</sup>This interpretation is in line with the following well-known result: under the CES, monopolistic competition emerges as the limit of a market involving a growing number of oligopolistic firms.



the mixed market must be cast within a general equilibrium frame to pin down the output growth effect stressed in Proposition 3.

(iv) It remains to determine the impact of an increase in the number of large firms on prices. It follows from (19) that  $\mathbf{P}$  and  $\mathbf{y}$  move in opposite directions. Proposition 5 therefore implies that  $\mathbf{P}$  decreases with  $N$ . Because  $M^*$  decreases with  $N$ , (14) implies that  $P_0$  increases. As a result, it must be that the equilibrium price  $P^*$  decreases with  $N$ . To sum-up, we have:

**Proposition 6** *In a mixed market, both the industry price index and the price at which the large firms sell their output decrease when the number of large firms increases.*

Thus, the addition of a large firm makes the whole market more competitive. Even though the exit of MC-firms tends to render the market less competitive (see Proposition 1), this effect is dominated by the pro-competitive effect generated by the expansion of big firms' output (see Proposition 3). This is reminiscent of what Basker (2007, p.195) writes about Wal-Mart, the entry of which has led the U.S. retail sector to become more effective "at providing consumers with the goods they want at better prices."

Furthermore, Proposition 5 implies that  $P^*(N) > P^*(\bar{N}) = P^O$  where  $P^O$  is the equilibrium price in the oligopoly case (see B.11), so that  $[P^*(N) - C]/P^*(N)$  is larger than  $(P^O - C)/P^O = 1 - \rho + \rho/N$ . Since this markup exceeds the markup under monopolistic competition  $1 - \rho$ , *the large firms' markup exceeds the small firms' markup*, which reflects the fact that the former have more market power than the latter. Thus, when both kinds of firms share the same marginal cost, the large firms price their varieties at a higher level than the small firms, thus confirming the above-mentioned idea that the small firms are more competitive than the big ones. Note, however, that the price ranking is reversed when the large firms have a sizable cost advantage.

Finally, because the entry of a large firm leads to a lower industry price index, combining (8) and Proposition 5 implies that the industry output index  $\mathbf{Q}$  increases. In other words, the decrease in the output index of the monopolistically competitive fringe is more than compensated by the expansion of the large firms' output. For this to arise, additional workers must be hired, which agrees with what Bertrand and Kramarz (2002) observe in the French retail industry.

Before concluding, it is worth noting that the foregoing analysis sheds light on some of the major trends characterizing the market dynamics in developed economies. Traditional economies were typically populated with small businesses and very few large firms. More affluent societies and technological progress have combined to facilitate the entry of a growing number of big firms. This in turn has triggered the decline of the small business subsector in mixed markets endowed with old and small firms as well as the growth of modern big firms (Mokyr, 2002). Eventually, when the number of large firms became sufficient large ( $N$  exceeds  $\bar{N}$ ), the monopolistically competitive fringe disappeared from the market.

However, our analysis also suggests that the fall in small firms' fixed costs sparked by the development of the new information technologies has permitted the revival of SMEs. Indeed, as predicted by our model, the launching of small firms became again profitable from the 1980s, which has led to the progressive emergence of new mixed markets. The evolution of markets, therefore, seems to be a non-monotone process, involving the transition from monopolistic competition to mixed markets through markets dominated by large oligopolistic firms. It is worth stressing that this discussion agrees with a well-documented fact stressed in the business literature on entrepreneurship, that is, the existence of a U-shaped relationship between the levels of entrepreneurship and economic development (see Wennekers *et al.*, 2010 for a survey and empirical evidence).

## 4 Welfare

The propositions derived in the above section open the door to welfare issues that we now investigate. Our purpose is not to conduct a first best analysis. Instead, we aim to determine whether or not the entry of a large firm is welfare-enhancing, which is precisely the question raised in political debates.

Because preferences (3) are homothetic, the level of social welfare may be described by the indirect utility corresponding to the utility of the representative consumer.<sup>9</sup> Substituting (6) and (7) into (3), we obtain the indirect utility:

$$W = \alpha^\alpha (1 - \alpha)^{1-\alpha} \mathbf{Y} \mathbf{P}^{-(1-\alpha)}. \quad (23)$$

When  $N$  increases, Proposition 5 implies that  $\mathbf{Y}$  increases whereas Proposition 6 tells us that  $\mathbf{P}$  goes down. Therefore, we have:

**Proposition 7** *In a mixed market, the entry of a large firm raises social welfare.*

In words, this result means that *a differentiated market with a few big firms and many small firms is less efficient than a market with more big firms and fewer small firms*. This runs against the conventional wisdom according to which a multitude of small firms does better in terms of social efficiency than a handful of large ones. This contrast in results is due to the fact that the mixed market model allows for direct comparisons of different market structures within a unified framework, thus shedding new light on their relative merits. It is also worth stressing that the above proposition is obtained in the case of a differentiated industry in which consumers have a preference for diversity. To be precise, Proposition 7 shows that the pro-competitive effect associated with the presence of

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<sup>9</sup>For the welfare analysis to be meaningful, preferences (3) are defined on the Cartesian product of (i) the vector space of dimension equal to the largest integer smaller than or equal to  $L/N$ , and (ii) the functional space of measurable functions defined on  $[0, L/f]$ .

large firms dominates the decrease in diversity generated by the exit of several small firms. We want to stress that Proposition 7 imposes no specific restriction on the parameters of the economy, apart from those stated in Proposition 2 that guarantee the existence of a mixed market equilibrium.

## 5 Concluding remarks

Mixed markets are plentiful in the real world, one reason being that keeping a monopolistically competitive fringe seems to be a political concern in several countries. Yet, our analysis suggests that consumers may gain from the presence of large firms because they render the market more competitive. Nevertheless, both in the public and the general press, it is customary to find the idea that the “small business” world of yesterday was more appealing than the “large business” world of today. Although sectors dominated by a few big firms were often more standardized than those involving many small producers, our analysis shows that consumers need not be better off under many small producers rather than under a handful of large ones. This is because the diversity argument put forward by interest groups ignores the pro-competitive effect that the entry of big firms brings about. Admittedly, our results are obtained in the case of a specific model, namely the CES. Being aware of its limits, we want to stress that this model is the workhorse of many contributions dealing with imperfect competition in modern economic theory. So our results cannot be dismissed on that basis only. Using a quadratic subutility nested into a linear utility, Parenti (2010) shows that the size of the monopolistically competitive fringe shrinks with the entry of a multiproduct firm. The same author also proves that the social surplus rises with the addition of a big firm (personal communication). Thus, our main results are robust against this alternative specification.

To conclude, observe that our setting can be applied to study various issues that have been investigated using the framework of monopolistic competition only. The first question that comes to mind is the opening to trade of two economies that have different mixed markets. Our analysis suggests that, by exacerbating competition between large firms, economic integration triggers the progressive disappearance of small firms. This would have the following important implication: if large firms have lower marginal costs than small firms, then trade liberalization would yield productivity gains in both countries. Second, it is worth studying the impact of large department stores or shopping malls that locate at the outskirts of a city, while competing with a large number of small shops located at the city center. In such a context, we conjecture that the exit of small shops make consumers living downtown worse-off when they have a bad access to the shopping malls.

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## Appendix A

We show that  $\mathbf{P}(M)$ ,  $P(M)$  and  $\pi^*(M)$  decrease with  $M$ . To simplify notation, we set

$$H \equiv \left( \frac{\rho \mathbf{P}}{c} \right)^{\frac{\rho}{1-\rho}} M. \quad (\text{A.1})$$

**Step 1.** Consider first the impact on  $\mathbf{P}(M)$  of increasing  $M$ . Substituting (13) into (17) and simplifying, we obtain:

$$\mathbf{y}^\rho = H \mathbf{y}^\rho + N \mathbf{P}^\rho Q^\rho.$$

Solving for  $Q$  yields

$$Q = \left( \frac{1-H}{N} \right)^{\frac{1}{\rho}} \mathbf{y} \mathbf{P}^{-1}. \quad (\text{A.2})$$

Substituting this expression into (18), we obtain

$$(N-1+H) \mathbf{P} = \frac{C}{\rho} N^{\frac{2\rho-1}{1-\rho}} (1-H)^{\frac{1-\rho}{\rho}}. \quad (\text{A.3})$$

Using (A.1), we may rewrite (A.3) as follows:

$$N^{\frac{1-2\rho}{1-\rho}} \frac{H(N-1+H)^{\frac{\rho}{1-\rho}}}{1-H} = \left( \frac{c}{C} \right)^{\frac{\rho}{1-\rho}} M. \quad (\text{A.4})$$

Define

$$G(H) \equiv N^{\frac{1-2\rho}{1-\rho}} \frac{H(N-1+H)^{\frac{\rho}{1-\rho}}}{1-H}$$

The function  $G$  increases with  $H$ , and is such that  $G(0) = 0$  and  $G(H) \rightarrow \infty$  when  $H \rightarrow 1$ . Therefore, for any given  $M$ , (A.4) has a unique solution  $H(M) \in ]0, 1[$ , which increases with  $M$ . It then follows from (A.3) that  $\mathbf{P}(M)$  decreases with  $M$ .

**Step 2.** Using (6) and (A.2), the equilibrium price set by a large firm is such that

$$P = N^{\frac{1-\rho}{\rho}} \mathbf{P} (1-H)^{\frac{1-\rho}{\rho}}. \quad (\text{A.5})$$

Because  $\mathbf{P}(M)$  decreases and  $H(M)$  increases with  $M$ , it must be that  $P(M)$  decreases with  $M$ .

**Step 3.** (A.5) implies that the profit of a large firm is given by

$$\Pi = \mathbf{y} N^{-1} (1-H) - C N^{-\frac{1}{\rho}} \mathbf{y} \mathbf{P}^{-1} (1-H)^{\frac{1}{\rho}} - F. \quad (\text{A.6})$$

Substituting (A.6) and (15) into (16) yields

$$\mathbf{y} \left[ 1 - \alpha + \alpha \rho H + \alpha C N^{-\frac{1-\rho}{\rho}} \mathbf{P}^{-1} (1 - H)^{\frac{1}{\rho}} \right] = \alpha (L - NF - Mf). \quad (\text{A.7})$$

Furthermore, it is readily verified that (A.3) is equivalent to

$$N^{-\frac{\rho}{1-\rho}} \mathbf{P}^{-1} (1 - H)^{\frac{1}{\rho}} = \left( \frac{C}{\rho} \right)^{\frac{-\rho}{1-\rho}} (N\mathbf{P})^{\frac{\rho}{1-\rho}} (N - 1 + H)^{\frac{1}{1-\rho}}.$$

Replacing in (A.7) yields the equilibrium income:

$$\mathbf{y}(M) = \frac{\alpha (L - NF - Mf)}{1 - \alpha + \alpha \rho H(M) + \alpha (C/\rho)^{\frac{-1}{1-\rho}} C (N\mathbf{P}(M))^{\frac{\rho}{1-\rho}} [N - 1 + H(M)]^{\frac{1}{1-\rho}}}. \quad (\text{A.8})$$

Substituting (A.8) into (15) yields

$$\pi^*(M) = \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} \frac{\alpha (1 - \rho) (L - NF - Mf)}{D(M)} - f \quad (\text{A.9})$$

where

$$D(M) \equiv (1 - \alpha) (\mathbf{P}(M))^{\frac{-\rho}{1-\rho}} + \alpha \rho \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} M + \alpha \left( \frac{C}{\rho} \right)^{\frac{-1}{1-\rho}} C N^{\frac{\rho}{1-\rho}} [N - 1 + H(M)]^{\frac{1}{1-\rho}}.$$

The numerator of (A.9) is decreasing in  $M$ , whereas the denominator  $D(M)$  is increasing because  $\mathbf{P}(M)$  decreases and  $H(M)$  increases with  $M$ . Consequently, for any  $N$  given the function  $\pi^*(M)$  must decrease with  $M$ .

## Appendix B

**Step 1.** We first determine a necessary and sufficient condition for a positive range of small firms to be in business in equilibrium.

Using  $H$  given by (A.1), we may rewrite the two equilibrium conditions (16) and (18) as follows:

$$L - NF - Mf = \mathbf{y} \left[ \frac{1 - \alpha}{\alpha} + \rho H + C N^{-\frac{1-\rho}{\rho}} (1 - H)^{\frac{1}{\rho}} \mathbf{P}^{-1} \right] \quad (\text{B.1})$$

$$1 = \left( \frac{C}{\rho} \right)^{-\frac{1-\rho}{\rho}} (1 - H)^{\frac{1-\rho}{\rho}} \mathbf{P}^{-1} + \frac{1 - H}{N}. \quad (\text{B.2})$$

Furthermore, we know from (15) that

$$\pi^* = \left( \frac{c}{\rho} \right)^{\frac{-\rho}{1-\rho}} (1 - \rho) \mathbf{y} \mathbf{P}^{\frac{\rho}{1-\rho}} - f.$$

Solving (B.1) for  $\mathbf{y}$ , (B.2) for  $\mathbf{P}$ , and substituting these expressions into  $\pi^*$  shows that

$$\text{sign } \pi^* = \text{sign } J$$

where

$$J(H) \equiv \left[ \frac{NC}{(N-1+H)c} \right]^{\frac{\rho}{1-\rho}} \frac{\alpha(1-\rho)(L-NF)(1-H)}{N[1-\alpha(1-\rho)(1-H)] - \alpha\rho(1-H)^2} - f$$

for  $H \in [0, 1]$ . The function  $J(H)$  is strictly decreasing and satisfies the border conditions

$$J(0) = \left[ \frac{NC}{(N-1)c} \right]^{\frac{\rho}{1-\rho}} \frac{\alpha(1-\rho)(L-NF)}{(1-\alpha)N + \alpha\rho(N-1)} - f \quad J(1) = -f.$$

Therefore, there exists a unique solution  $H^* \in ]0, 1[$  to  $J(H) = 0$  if and only if  $J(0) > 0$ . This condition is equivalent to assuming that

$$\frac{f}{L} < S\left(N; \frac{F}{L}\right) \equiv \left[ \frac{NC}{(N-1)c} \right]^{\frac{\rho}{1-\rho}} \frac{\alpha(1-\rho)}{(1-\alpha)N + \alpha\rho(N-1)} \left(1 - \frac{NF}{L}\right). \quad (\text{B.3})$$

Thus, there is a monopolistically competitive fringe if and only if this condition holds.

**Step 2.** We now show that there exists a unique mixed market equilibrium.

Using (6), the profit of a large firm evaluated at a symmetric outcome is given by

$$\Pi(Q) = \mathbf{y}^{1-\rho} Q^\rho \mathbf{P}^\rho - CQ - F. \quad (\text{B.4})$$

The zero-profit condition (19) may be rewritten as follows:

$$\mathbf{y}^{1-\rho} \mathbf{P}^\rho = \gamma \quad (\text{B.5})$$

where

$$\gamma \equiv \left(\frac{c}{\rho}\right)^\rho \left(\frac{f}{1-\rho}\right)^{1-\rho} > 0.$$

Substituting (B.5) into (B.4), we obtain

$$\Pi = \gamma Q^\rho - CQ - F$$

and thus

$$\mathbf{Y} = N(\gamma Q^\rho - CQ - F) + L. \quad (\text{B.6})$$

Equating (22) and (B.6) shows that the equilibrium output  $Q^*$  solves the equation

$$\Delta(Q) - \Gamma(Q) = 0 \quad (\text{B.7})$$

where

$$\begin{aligned} \Delta(Q) &\equiv N\Pi(Q) = N(\gamma Q^\rho - CQ - F) \\ \Gamma(Q) &\equiv F(Q) - L = \frac{\gamma}{\alpha} \frac{Q^\rho}{1 - (C/\gamma\rho)Q^{1-\rho}} - L \end{aligned}$$



which are both increasing in  $Q$  over the interval  $[0, \bar{Q}]$  where

$$\bar{Q} \equiv \left( \frac{C}{\gamma\rho} \right)^{\frac{-1}{1-\rho}} > 0.$$

Observe that  $\Delta(Q)$  increases with  $N$  whereas  $\Gamma(Q)$  is independent of  $N$  (see Figure 2).

It is readily verified that (i)  $\Delta(Q)$  is concave and increasing, (ii)  $\Gamma(Q)$  is convex and increasing, with  $\Delta(0) - \Gamma(0) = L - NF > 0$ , and (iii)  $\Delta(Q) - \Gamma(Q)$  tends to  $-\infty$  when  $Q = \bar{Q}$ . As a consequence, (B.7) has a unique positive solution  $Q^*$ . Note that  $Q^*$  is smaller than  $\bar{Q}$  because  $\Gamma(Q)$  tends to  $\infty$  at  $\bar{Q}$ .

**Step 3.** It remains to find a necessary and sufficient condition for the large firms' profits to be positive at  $Q^*$ , that is, a condition for  $\Delta(Q^*) > 0$  to hold.

It follows from the properties of  $\Delta$  and  $\Gamma$  that

$$\begin{aligned} \Delta(Q) &> \Gamma(Q) &\Leftrightarrow & Q < Q^* \\ \Delta(Q) &= \Gamma(Q) &\Leftrightarrow & Q = Q^* \\ \Delta(Q) &< \Gamma(Q) &\Leftrightarrow & Q > Q^*. \end{aligned}$$

Because  $\Gamma(0) = -L$  and  $\Gamma(Q) \rightarrow \infty$  when  $Q \rightarrow \bar{Q}$ ,  $\Gamma(Q) = 0$  has a unique solution  $Q_s \in ]0, \bar{Q}[$ . The comparison of  $Q^*$  and  $Q_s$  involves the following three cases.

(a) If  $\Delta(Q_s) < 0$ , then  $\Delta(Q_s) < \Gamma(Q_s) = 0$ , so that  $Q_s > Q^*$ . Since  $\Delta(Q)$  is increasing, we have  $\Delta(Q^*) < \Delta(Q_s) < 0$ .

(b) If  $\Delta(Q_s) = 0$ , then  $\Delta(Q_s) = \Gamma(Q_s) = 0$ . Since (B.7) has a unique positive solution, it must be that  $Q_s = Q^*$ . In this case, we have  $\Delta(Q^*) = 0$ .

(c) If  $\Delta(Q_s) > 0$ , then  $\Delta(Q^*) > \Gamma(Q_s) = 0$ , and thus  $Q_s < Q^*$ . Since  $\Delta(Q)$  is increasing, it must be that  $\Delta(Q^*) > \Delta(Q_s) > 0$ .

Hence, we have:

$$\Delta(Q^*) > 0 \quad \Leftrightarrow \quad \Delta(Q_s) > 0.$$

Consequently, the large firms' profits are positive if and only if  $\Delta(Q_s) > 0$ .

Set

$$G(Q^\rho) \equiv \gamma^2 \rho (Q^\rho)^2 + \alpha(1 - \rho)L\gamma Q^\rho - \alpha LF$$

and observe that

$$\Gamma(Q_s) = 0 \Leftrightarrow -CQ_s = \frac{\gamma^2}{(1 - \alpha)\rho L} (Q_s^\rho)^2 - \gamma Q_s^\rho.$$

Plugging  $-CQ_s$  into  $\Delta(Q_s)$ , it is readily verified that  $\Delta(Q_s) > 0$  is equivalent to  $G(Q_s^\rho) > 0$ . This inequality holds if and only if  $Q_s^\rho > Q_r^\rho$  where

$$Q_r^\rho \equiv \frac{\alpha(1-\rho)L}{2\gamma\rho} \left[ \sqrt{\frac{4\rho F}{\alpha(1-\rho)^2 L} + 1} - 1 \right] \quad (\text{B.8})$$

is the positive root of the quadratic equation  $G(Q^\rho) = 0$ .

Since  $\Gamma(Q)$  is increasing and  $\Gamma(Q_s) = 0$ , we have:

$$Q_r^\rho > Q_s^\rho \Leftrightarrow \Gamma(Q_r) < 0 \Leftrightarrow \gamma Q_r^\rho \left( 1 - \frac{\gamma}{\alpha L} Q_r^\rho \right) > \frac{C}{\rho} Q_r.$$

Substituting (B.8) into the last inequality, we obtain the desired condition:

$$\begin{aligned} Q_r < Q_s \Leftrightarrow B\left(\frac{F}{L}\right) &\equiv \left[ \frac{\alpha(1-\rho)}{2} \right]^{\frac{1}{1-\rho}} \left[ \frac{\rho}{\alpha\rho - F/L} \frac{C}{c} \right]^{\frac{\rho}{1-\rho}} \\ &\cdot \left[ \frac{2}{\alpha(1-\rho)} \frac{F}{L} - 1 + \sqrt{\frac{4\rho}{\alpha(1-\rho)^2} \frac{F}{L} + 1} \right] < \frac{f}{L}. \end{aligned} \quad (\text{B.9})$$

To sum-up, the mixed market equilibrium exists and is unique if and only if (B.3) and (B.9) hold. This equilibrium is implicitly given by

$$\begin{aligned} P^* &= \frac{C}{\rho} \frac{N(1-H)^{\frac{2(1-\rho)}{\rho}}}{N-1+H} \\ Q^* &= \frac{\rho(1-\alpha)(N-1+H)(1-H)(L-NF-Mf)}{CN \{ \alpha N + (1-\alpha)\rho [N - (1-H)^2] \}} \\ M^* &= \left( \frac{c}{C} \right)^{\frac{\rho}{1-\rho}} \frac{NH}{1-H} \left( \frac{N-1+H}{N} \right)^{\frac{\rho}{1-\rho}} \\ \mathbf{Y}^* &= \frac{N(L-NF-Mf)}{(1-\alpha)N + \alpha\rho [N - (1-H)^2]}. \end{aligned} \quad (\text{B.10})$$

**Step 4.** Observe, finally, that the market involves a pure oligopoly when  $M^*(N) = 0$ . If  $J(0) \leq 0$ , then  $\pi^*(H) < 0$  for all  $H > 0$ . In this case, the oligopoly market outcome is obtained by setting  $H = 0$  in the expressions (B.10). This yields the following equilibrium values, which all decrease with  $N$ :

$$P^O = \frac{C}{\rho} \frac{N}{N-1} \quad (\text{B.11})$$

$$Q^O = \frac{\alpha\rho(N-1)(L-NF)}{CN [(1-\alpha)N + \alpha\rho(N-1)]} \quad (\text{B.12})$$

$$\Pi^O = \frac{\alpha [(1-\rho)N + \rho] (L-NF)}{N [(1-\alpha)N + \alpha\rho(N-1)]} - F$$

$$\mathbf{Y}^O = \frac{N(L-NF)}{(1-\alpha)N + \alpha\rho(N-1)}.$$

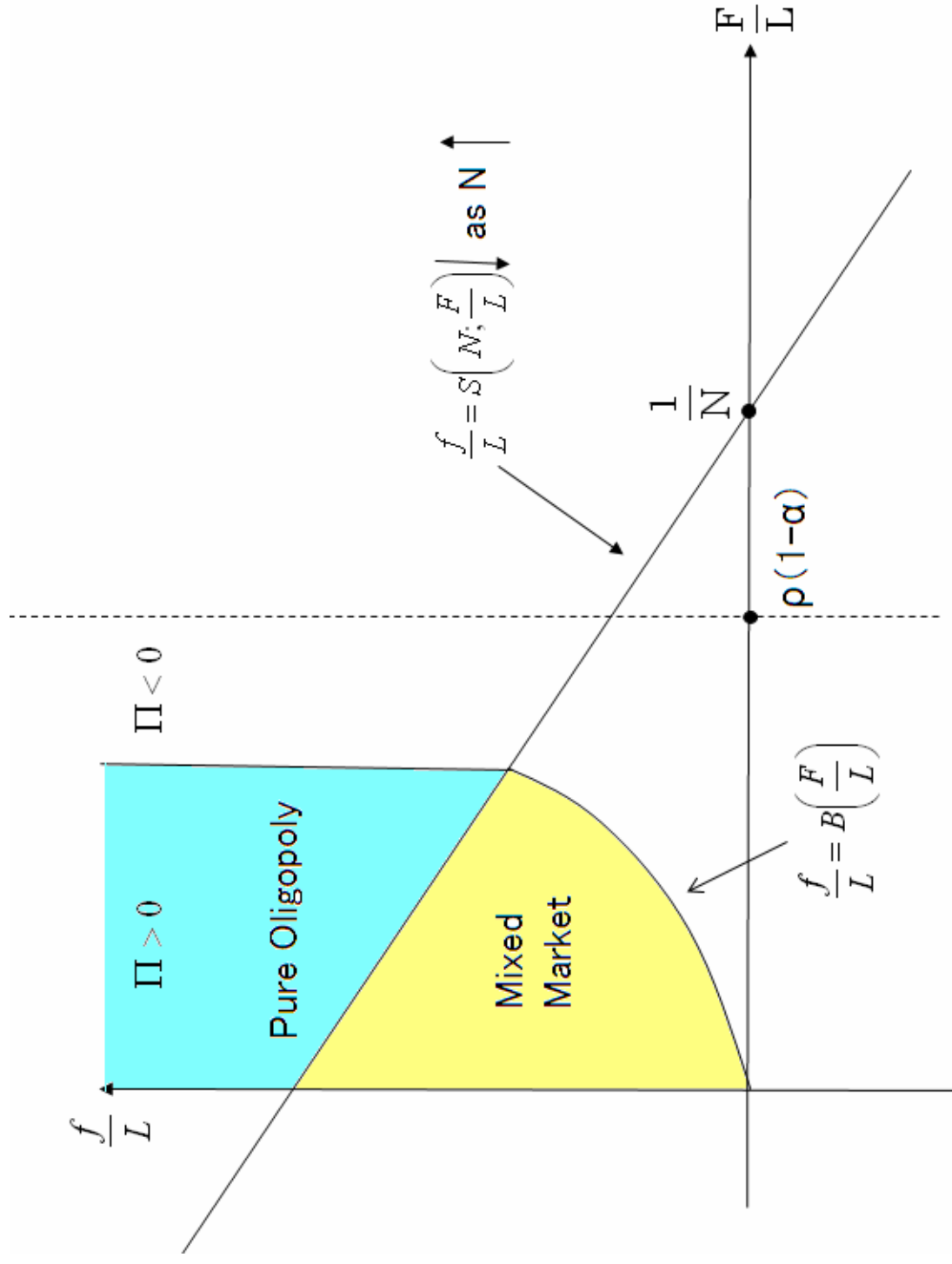


FIGURE 1: The emergence of mixed market



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